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# FOREIGN TECHNOLOGY DIVISION

METHODS OF GUIDANCE FLIGHT DYNAMICS

by

Zhu Wenxuan



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# METHODS OF GUIDANCE FLIGHT DYMANICS

Zhu Wenxuan

Translation of "Zhi Dao Fei Xing Li Xue Fang Fa"; *Journal of Astronautics*, No.4; OCT, 1989

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SUMMARY Setting up and solving ballistic type rocket kinetic equations in inertial coordinate systems with directions which do not vary, it is possible not to consider dragging inertial or centrifugal forces or coriolis inertial forces and set up inertial guidance on a convenient and simple, quick and clear foundation. This is different from what has been published in the past, where studies have been done of various types of flight mechanics problems in coordinate systems which turn, following along with the earth. We illucidate the initial take off conditions as rockets leave the surface of the earth, and analyze, in detail, the air or atmospheric effects associated with air being pulled along by the gravity of the earth as it turns and the treatment methods for aerodynamic forces on rockets moving in an inertial space. We derive a formula for calculating the geographical location of movement parameters, used for rockets in inertial coordinate systems, and a formula for dynamic coordinate system movement parameters.

KEY TERMS Inertial Guidance, Coordinates, Flight Mechanics, Kinetic Equation

As far as the literature[1,2,3,4] and publications associated with the kinetics and kinematics of domestic and foreign studies of lifting rockets and other similar types of spacecraft are concerned, although all start out through classical mechanics, in the end, however, they all infer the handling of aerodynamic forces in coordinate systems which turn together along with the earth as well as various similar types of forces and motion parameters. Moreover, it is necessary to consider centrifugal inertial forces and coriolis inertial forces as well as other similar types of virtual forces. Speaking in terms of the researchers who have handled guidance and navigation systems for rockets and other similar types of spacecraft, if one analyzes motion characteristics of inertial instruments, it becomes even more complicated. In many years of research and practical application, we took rocket kinetics problems and solved them in an unmoving inertial space, obtaining satisfactory results.

# I. SEVERAL COORDINATE SYSTEMS[3]

1. The Earth Centered Coordinate System $E, X_e, Y_e, Z_e$.

The origin point E is placed at the geometrical center of the earth. The $EX_e$ axis in the plane of the equator, from the center of the earth E, points toward the point of intersection of the prime meridian and the equator and is positive in direction. The $EY_e$ axis, in the plane of the equator, is perpendicular to the $EX_e$ axis and is positive in direction toward the outside. The $EZ_e$ axis, along the axis of the earth's autorotation, forms a clockwise orthogonal coordinate system with $EX_e$ and $EY_e$. This is as shown in Fig.1.

2. Guidance Coordinate System $O_o, X_o, Y_o, Z_o$

The origin point $O_o$ is the launch point of the rocket on the earth's surface. The $O_oX_o$ axis, within the horizontal plane that passes through $O_o$, points toward the direction of the flight and is positive. The included angle or angle of offset between it and the due north direction of the meridian group tangent is the azimuth angle. The $O_oY_o$ axis follows along a plumbline upward and is positive. The $O_oZ_o$ axis, in the horizontal plane that passes through $O_o$, forms a clockwise orthogonal coordinate system with $O_oX_o$ and $O_oY_o$. At the instant that the rocket leaves the launch pad (time t=0sec), one takes the direction $O_oX_oY_oZ_o$ is pointing in inertial space and firmly fixes it invariable. It then forms an inertial coordinate system. See Fig.1.

3. Launch Coordinate System or Dynamic Coordinate System OXYZ.

At the instant the rocket takes off ($t=0^s$), the origin points O and $O_o$, the OX axis and $O_oX_o$, the OY axis and $O_oY_o$, and the OZ axis and $O_oZ_o$, are all superimposed on each other. After the rocket leaves the ground and takes off, OXYZ is firmly connected to the surface of the earth. Moreover, it turns together with the earth. See Fig.1. 31

## 4. Spacecraft Coordinate System $O_1X_1Y_1Z_1$

The origin point $O_1$ is positioned at the center of mass of the rocket body. The $O_1X_1$ axis runs along the main axis of inertia of the rocket, points toward the nose, and is positive. $O_1Z_1$, in the secondary plane, is perpendicular to the $X_1O_1Y_1$ main plane of symmetry. This is as shown in Fig.2.

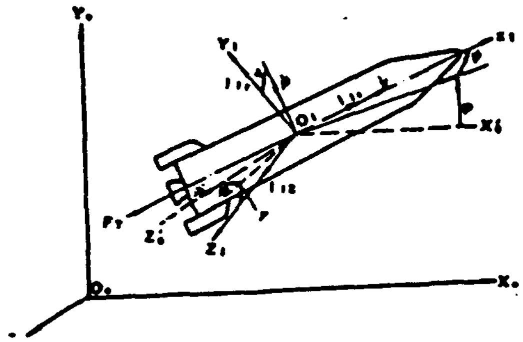

Fig.1 The Relationships of Such Coordinate Systems as $EX_eY_eZ_e$, $OX_oY_oZ_o$, and OXYZ

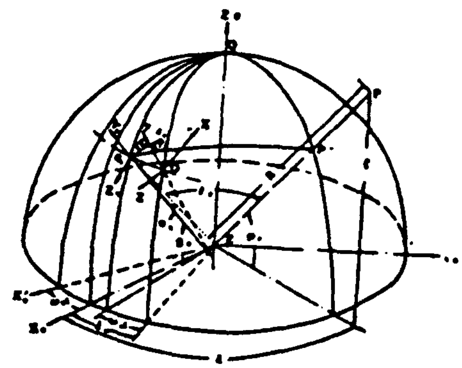

Fig.2 The Relationship of $O_oX_oY_oZ_o$ and $O_1X_1Y_1Z_1$

5. Airflow Coordinate System $O_2X_2Y_2Z_2$

The origin point $O_2$ is conincident with the center of mass of the rocket body $O_1$. The axis $O_2X_2$ is in line with the air flow velocity vector $V_d$ corresponding to the rocket. The $O_2Y_2$ axis, in the main rocket plane $X_1O_1Y_1$, goes upward and is positive. The $O_2Z_2$ axis forms a clockwise rectangular coordinate system with the $O_2X_2$ and $O_2Y_2$ axses. If one takes the $O_2X_2Y_2Z_2$ coordinate system and rotates it about the $O_2Y_2$ axis through the lateral slide angle $\beta$, and, again, rotates it about the $O_1Z_1$ axis through the angle of attack a, then it is coincident with O,X,Y,Z. See Fig.3.

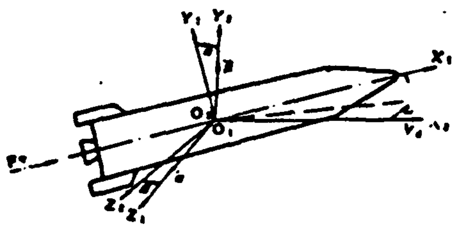

Fig.3 The Relationships of $O_1, X_1, Y_1, Z_1$ and $O_2, X_2, Y_2, Z_2$

The unit vectors on the axses of each coordinate system are taken as a unit 1, adding the corresponding coordinate system's lower corner subscripts to express that. For example, $1_{ox}$, $1_{oy}$, and $1_{oz}$ are the unit vectors of the guidance or inertial coordinate system $O_oX_oY_oZ_o$ on the three axses $O_oX_o$, $O_oY_o$, and $O_oZ_o$.

## II. KINETIC EQUATIONS

Taking out the influences of other celestial bodies, one only considers the effects of earth's gravity, $G_e$. The high speed flow of combustion gases sprayed out by the rocket engines produces as an effect the thrust vector $F_T$ on the rocket. Its magnitude is related

to the amount of mass consumed as fuel in a unit time $\dot{G}=dG/dt$ (kg/s), the engine jet tube characteristics, and the height h at which the rocket flies off the surface. Normally, this can be expressed as[1,2,3]

$$F_T=\dot{G}\cdot g_e\cdot I_b+S_eP_e\left(1-\frac{P}{P_e}\right) \qquad (2.1)$$

On the basis of the principles of momentum, the rocket's instantaneous acceleration vector in the inertial coordinate system $O_oX_oY_oZ_o$, $\dot{V}_o=dV_o/dt$, satisfies the equation below

$$G(t)\cdot\dot{V}_o=F+G_o, \qquad (2.2)$$

Force vector F is the vector sum of thrust $F_T$, control force $F_K$, and the pneumatic force $F_q$. $g_e=9.80665$ (m/s$^2$) and is the mass conversion constant. $I_b$ is the engine's specific impulse. $S_{e(\text{unclear})}$ is the engine combustion gas flow aperture area. $P/P_o$ is the ratio of the gas pressure and the surface atmospheric pressure $P_o$, which varies with the height h from which rockets fly off the surface. If one takes a certain vector, such as F, and uses a line matrix to express it, its symbols and the vector symbols are the same. One then has $F=[F_XF_YF_Z]^T$. The upper right superscript T expresses transposed positions. $F_X$, $F_Y$, and $F_Z$ are the components of F on the $O_oX_o$, $O_oY_o$, and $O_oZ_o$ axses. When using the acceleration of gravity $g=[g_Xg_Yg_Z]^T$ and comparative force or perveived acceleration $\dot{W}=[\dot{W}_X\dot{W}_Y\dot{W}_Z]^T$ to express equation (2.2), one obtains 32

$$\dot{V}_o=\dot{W}+g \qquad (2.3)$$

In this 其中 $g=\frac{1}{G}G_o$, $\dot{W}=\frac{1}{G}F$. . The force vector

$$F=F_T+F_K+F_q \qquad (2.3.1)$$

The thrust force $F_T$ and control force $F_K$ are normally given in cubic coordinate systems. Moreover, the aerodynamic force $Q_2$ in the gas flow coordinate system $O_2X_2Y_2Z_2$ can be transformed into $O_1X_1Y_1Z_1$ and expressed. In this way, the comparative force components on the main inertial $O_1X_1$, $O_1Y_1$, and $O_1Z_1$ axses are respectively

$$\dot{W}_{X1}=\frac{1}{G}(F_{1TX}+F_{1KX}+Q_{1X}) \tag{2.4.1}$$

$$\dot{W}_{Y1}=\frac{1}{G}(F_{1TY}+F_{1KY}+Q_{1Y}) \tag{2.4.2}$$

$$\dot{W}_{Z1}=\frac{1}{G}(F_{1TZ}+F_{1KZ}+Q_{1Z}) \tag{2.4.3}$$

The comparative force vector $\dot{W}_1=[\dot{W}_{X1}\dot{W}_{Y1}\dot{W}_{Z1}]^T$. The relationship between the unit vectors $1_O$ and $1_1$ of $O_oX_oY_oZ_o$ and $O_1X_1Y_1Z_1$ can be used to link up or associate the orthogonal transformation matrix D, which is nothing else than

$$1_0=D\cdot 1_1 \tag{2.5}$$

Because of this, the comparative force vector W in the guidance coordinate system $O_oX_oY_oZ_o$ is capable of being aided by $D_1$ using the comparative force vector $\dot{W}_1$ in $O_1X_1Y_1Z_1$ to express as

$$\dot{W}=D\cdot\dot{W}_1 \tag{2.6}$$

Gauging or measurement assemblies used in the guidance and stability systems of such types of carrier or delivery craft as rockets are, it goes without saying, gyroscope stabilized platforms or sensitively connected inertial assemblies. The various elements of the orthogonal transformation matrix D, $d_{ij}$ (i=1,2,3, j=1,2,3), are all capable of being arrived at. For example, in Fig.2, rotating the pitch angle $\varphi$ around the axis $O_oZ_o$, and again, rotating the yaw angle $\psi$ around the transitional axis $OY'_1$, and, finally, revolving the roll angle $\gamma$ around the axis $O_1X_1$, in that case, $d_{ij}$ is then possible to make use of to obtain the sine and cosine functions of Euler angles $\varphi$, $\psi$, and $\gamma$.

Taking rockets, when they act as rigid bodies, the Euler equations for the revolved center of mass $O_1$ are normally set up in a cubic coordinate system[6]:

$$J_X\dot{\omega}_{1X}+(J_Z-J_Y)\omega_{1Z}\omega_{1Y}=M_{1X} \tag{2.7.1}$$

$$J_Y\dot{\omega}_{1Y}+(J_X-J_Z)\omega_{1X}\omega_{1Z}=M_{1Y} \tag{2.7.2}$$

$$J_Z\dot{\omega}_{1Z}+(J_Y-J_X)\omega_{1Y}\omega_{1X}=M_{1Z} \tag{2.7.3}$$

In the equations, $J_X$, $J_Y$, and $J_Z$ are the instantaneous momenta of rotation of the rocket about the main axses of momentum $O_1X_1$, $O_1Y_1$, and $O_1Z_1$. $M_{1X}$, $M_{1Y}$, $M_{1Z}$ are component moments of force along the three axses discussed above. $\omega_{1X}$, $\omega_{1Y}$, $\omega_{1Z}$ are components of angular acceleration for rotation of the rocket about $O_1X_1$, $O_1Y_1$, and $O_1Z_1$. The angular acceleration vector is

$$\omega_1=[\omega_{1X}\ \omega_{1Y}\ \omega_{1Z}]^T \tag{2.8}$$

This article only illucidates movements of the center of mass. It does not discuss the kinetic equations for motions around center of mass.

## III. INITIAL CONDITIONS AND AERODYMANIC FORCES

In the inertial coordinate system $O_oX_oY_oZ_o$, as far as study of the flight kinetics of rockets and other similar carrier or delivery vehicles is concerned, the most important thing is to illucidate the methods of handling the effects of aerodynamic forces with which the atmosphere acts on rockets in high speed flight as it rotates along with the earth, and, on the foundation of discussions with fellow workers, in conjunction with the carrying out of theoretical analysis, to obtain correct solutions.

Observed in inertial space, rockets, before they take off, rotate together along with the earth. They are dragged along by the force of the earth's gravity and connected firmly to the surface of the earth. However, they already possess a drawing or centrifugal linear acceleration. This velocity vector $V_{oo}$ is the initial velocity taking off from earth under the effect of the thrust of the engines. The magnitude of $V_{oo}$ is determined by the vertical distance of the launch point $O_o$ from the earth's axis of autorotation. In the same way, the atmosphere, which wraps around the earth, is also pulled firmly by the force of the earth's gravity and follows the earth in its rotation. Observed from inertial space, this atmosphere also possesses similar linear acceleration.

Looking from the revolving earth, the rocket, before it takes off, is not moving. The atmosphere is stationary. The rocket's movement velocity relative to the atmosphere is zero. As a result of this, there are no aerodymanic or pneumatic force effects on the stationary rocket. This is saying nothing else than that, if, when one calculates the magnitude of the effects of aerodynamic or pneumatic forces on the rocket in inertial space, it is necessary to take the dragging or centrifugal velocity associated with the rocket rotating along with the earth and the velocity of the atmosphere moving together with the earth and subtract them from each other, one obtains the rocket's velocity relative to atmospheric movements as zero, making the aerodymanic forces borne by the rocket, when it is erect on the launch pad and has not taken off yet, to be zero. If one takes the initial absolute velocity $V_{oo}$ of the rocket before it takes off, as observed in inertial space, and takes it as the movement velocity relative to the atmoshere, calculating out the effects of

pneumatic forces on the rocket, in that case, the rocket erected on the launch pad will just be pushed over onto the ground. However, this does not correspond to the facts.

After the rocket, under the effects of thrust $F_T$, leaves the launch pad and takes off, the absolute velocity of the motion in inertial space gets greater and greater. The height of the separation from the earth's surface is also able to continuously increase. The magnitude of the aerodynamic forces that it exerts at a given point P separated from the center of the earth E is also determined by the velocity of the atmosphere corresponding to its being pulled around by the earth. It is only in the inertial space $O_oX_oY_oZ_o$ that, as far as the rocket's velocity relative to the atmosphere is concerned, it is appropriate to use this relative air flow velocity to calculate the effects of pneumenatic or aerodynmaic forces on the rocket.

The components of the earth's atuorotational velocity vector $\omega_e$ on the various individual axses of the inertial or guidance coordinate system $O_oX_oY_oZ_o$ are:

$$\Omega_X=\omega_e\cdot\cos B_o\cdot\cos A_o=\omega_e b_X \qquad (3.1.1)$$

$$\Omega_Y=\omega_e\cdot\sin B_o=\omega_e b_Y \qquad (3.1.2)$$

$$\Omega_Z=\omega_e\cdot(-\cos B_o\cdot\sin A_o)=\omega_e\cdot b_Z \qquad (3.1.3)$$

In these, $B_O$ is the geographical latitude of the rocket launch point $O_O$. $A_O$ is the azimuth angle of $O_OX_O$. In $O_OX_OY_OZ_O$, the absolute velocity of the air at the place P(R) where the rocket is located is nothing else than the dragging or centrifugal velocity $V_e$. It can be expressed as

$$V_e=\omega_e\times R=\begin{bmatrix}1_{eX} & 1_{eY} & 1_{eZ}\\ \Omega_X & \Omega_Y & \Omega_Z\\ R_X & R_Y & R_Z\end{bmatrix} \qquad (3.2)$$

The magnitude of the radius vector from the point P toward the center of the earth, $R=[R_X R_Y R_Z]^T$, is

$$R=\sqrt{(X_0+R_{0x})^2+(Y_0+R_{0y})^2+(Z_0+R_{0z})^2} \qquad (3.3)$$

In this, $R_1=[R_{OX}, R_{OY}, R_{OZ}]^T$ is the radius vector toward the center of the earth for the initial location of the rocket $O_O$ in $O_O X_O Y_O Z_O$. From the origin point $O_O$, the position vector pointing toward P is $\xi = [X_O Y_O Z_O]^T$.

Given initial conditions, that is to say, the instant of rocket take off is t=0sec and the location relative to the $O_O X_O Y_O Z_O$ system is zero, that is, $X_O=Y_O=Z_O=0$ and $R=R_1$, the initial velocity $V_{OO}$ is nothing else than the dragging or centrifugal velocity of point $O_O$, that is, 34

$$V_{01}=V_0|_{t=0}=V_e|_{t=0} \qquad (3.4)$$

In this way, the variable coefficient differential equation (2.3) is then capable of using numerical value methods for its solution. The absolute velocity of the rocket $V_O$ at a certain instant as well as the position $\xi$ and R can all be obtained.

The rocket's velocity relative to the motion of the earth is also nothing else than the motion velocity $V_d$ relative to the atmosphere and is the difference between the absolute velocity $V_O$ and the dragging or centrifugal velocity $V_{e(unclear)}$

$$V_d=V_0-V_e \qquad (3.5)$$

The aerodynamic drag force $X_q$ and the relative velocity $V_d$ are mutually opposite in direction, that is, they are mutually opposite to the direction of $O_2X_2$. The areodynamic or pneumatic lift (normal pneumatic lift) $Y_q$ is positive in the direction along $O_2Y_2$. The lateral pneumatic force $Z_q$ is opposite in direction to $O_2Z_2$. They are respectively used in the several forms set out below for calculations

$$X_{q} = -C_x \cdot q \cdot S_c \qquad (3.6)$$

$$Y_{q} = C_y^{\alpha} \cdot q \cdot S_c \cdot \alpha \qquad (3.7)$$

$$Z_{q} = -C_z^{\beta} \cdot q \cdot S_c \cdot \beta \qquad (3.8)$$

In this, $C_X$, $C_Y^{\alpha}$, and $C_{z(\text{unclear})}^{\beta}$ are, respectively, the aerodymanic or pneumatic drag coefficient of the rocket, the aerodynamic lift coefficient, and the aerodynamic or pneumatic laterally directed force coefficient. As far as impact pressure is concerned, one uses the form

$$q = \frac{1}{2} \rho \cdot V_{\kappa}^{2} \qquad (3.9)$$

to calculate it. The air density $\rho$ changes along with changes in the height h of the rocket as it flies off the surface. $S_C$ is the reference cross section surface area.

From Fig.3 one can see that the airflow coordinate system $O_2X_2Y_2Z_2$ goes through a rotation through the lateral slide angle $\beta$ and the angle of attack a to arrive at $O_1X_1Y_1Z_1$. The transformation between the unit vectors of the two coordinate systems uses the matrix E and is expressed as

$$1_1 = E \cdot 1_2 \qquad (3.10)$$

Obviously, E is a third order orthogonal matrix. Its various elements are functions of a and $\beta$ :

$$E = \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{bmatrix} = \begin{bmatrix} \cos\beta\cdot\cos\alpha & \sin\alpha & -\sin\beta\cdot\cos\alpha \\ -\cos\beta\cdot\sin\alpha & \cos\alpha & \sin\beta\cdot\sin\alpha \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \qquad (3.11)$$

The angle of attack a and the lateral slide angle $\beta$ are calculated by the use of the formulae set out below:

$$\alpha = \sin^{-1}(-V_{1y}/V_1) \tag{3.12}$$

$$\beta = \sin^{-1}(V_{1z}/\sqrt{V_{1x}^2+V_{1z}^2}) \tag{3.13}$$

In the cubic coordinate system $O_1X_1Y_1Z_1$, the relative velocity $V_1=[V_{1X}V_{1Y}V_{1Z}]^T$ is obtained by $V_d$ in the $O_oX_oY_oZ_o$ system going through transformation. It is capable of being expressed as

$$V_1 = D^T V_d \tag{3.13}$$

Obviously, $V_1 = V_d$ are invariant. As far as the aerodynamic or pneumatic force $Q_2=[X_q Y_q Z_q]^T$ in the airflow coordinate system $O_2X_2Y_2Z_2$ is concerned, it is possible to make use of the matrix E to transform it into the cubic coordinate system $O_1X_1Y_1Z_1$, that is,

$$Q_1 = E \cdot Q_2 \tag{3.14}$$

Control force vectors $F_K$ are always given in the $O_1X_1Y_1Z_1$ coordinate system. Practically speaking, their handling is different because of different types of servomechanisms. Up to now, we have already, in $O_1X_1Y_1Z_1$, completely obtained the thrust force $F_{1T}$, the aerodynamic or pneumatic force $Q_1$, and the control force $F_K$. It is possible, from equations (2.4) and (2.6), to calculate out the comparative forces for changes over time, W and $W_{t(unclear)}$.

When one takes the earth to be acting as a revolving, symmetrical ellipsoid sphere, it is possible to only give consideration to the gravitaional force potential's second degree harmonic coefficient $J_2$ quantity[5]. The components of the gravitational acceleration g, which the rocket receives pointing toward the vicinity of the center

of the earth and which are in directions along $O_oX_o$, $O_oY_o$, and $O_oZ_o$ are:

$$g_v = g_c \cdot \left\{ -\frac{R_c^2}{R^2} \cdot \frac{R_v}{R} + J_2 \frac{R_c^4}{R^4} \cdot \left[ \frac{R_v}{R} \cdot \left( 5\frac{\zeta^2}{R^2} - 1 \right) - 2b_v \frac{\zeta}{R} \right] \right\} \qquad (3.15)$$
$$(v = X, Y, Z)$$

In this, the constant of gravitational force is

$$g_c \cdot R_c^2 = fM, \qquad (3.16)$$

$R_C$ is the earth's equatorial radius. $g_C$ is the acceleration of gravity on the equator. The vertical distance that the rocket's real time position P(R) is off the plane of the equator is

$$\zeta = b_x(X_0 + R_{0x}) + b_y \cdot (Y_0 + R_{0y}) + b_z \cdot (Z_0 + R_{0z}) \qquad (3.20)$$

The point P's geocentric latitude $\varphi_x$ is calculated from the equation

$$\varphi_x = \sin^{-1}\left(\frac{\zeta}{R}\right) \qquad (3.21)$$

$2b_v \sin\varphi_x$ in $g_v$ does not exist in OXYZ when one is studying the motion equations for the center of mass of rockets.

The application of the various expressions set out above is a condition for the complete solution of the kinetic equation (2.3). It is then possible to precisely determine, in changing situations, such motion parameters as the absolute velocity $V_o$ and the position R or $\xi$

## IV. KINETIC EQUATIONS FOR THE UNPOWERED SECTION

As far as the rocket or its released nose cone is concerned, in

the section of the flight after the propulsion system is shut off and there are no thrust force effects or attitude angle controls, it is possible to take them to be acting as a point mass or material point particle. It is not necessary to consider the angle of attack a or the lateral slide angle $\beta$. The only aerodynamic force there is is drag. Moreover, it is opposite in direction to $V_d$,

$$X_t = -C_{xt} \cdot q \cdot S_t \tag{4.1}$$

The comparative force is

$$W_b = \frac{S_t}{2G_t} \cdot C_{xt} \cdot \rho \cdot V_d^2 \tag{4.2}$$

In this, $C_{xt}$, $S_t$, and $G_t$ are, respectively, the nose cone's aerodynamic drag coefficient, reference cross section surface area, and its mass. Taking $W_b$ and analyzing it along the three axses of the inertial coordinate system $O_oX_oY_oZ_o$, one then has kinetic equations similar to (2.3);

$$\dot{V}_{ox} = -W_b \frac{V_{dx}}{V_d} + g_x \tag{4.3.1}$$

$$\dot{V}_{oy} = -W_b \frac{V_{dy}}{V_d} + g_y \tag{4.3.2}$$

$$\dot{V}_{oz} = -W_b \frac{V_{dz}}{V_d} + g_z \tag{4.3.3}$$

In this, the form expressing the acceleration of gravity $g=[g_x g_y g_z]^T$ is the same as (3.15). The initial conditions for equation (4.3) are the motion parameters $V_o(tf)$ and $\xi$ (tf) for the instant when the thrust stops or the instant tf when the nose cone and the rocket separate.

In situations in which the flight of the nose cone reentering the atmosphere has attitude angle controls, the aerodynamic force $\varsigma_2$ contains the drag force $X_q$, the lift force $Y_q$, and the lateral force $Z_q$. It is necessary to make use of the handling methods of 3. Moreover, one must do transformations into the coordinate system (for example, $O_5X_5Y_5Z_5$) firmly connected to the "horse's head" cone. After solving Euler equations similar to (2.7), one obtains the transformation matrix D. In this way, the aerodynamic or pneumatic relative forces are also capable of being accurately handled in $O_oX_oY_oZ_o$.

## V. MOTION PARAMETERS OF THE REVOLVING EARTH

The preceding several sections studied methods of handling aerodynamic forces in inertial space and the problems of solving kinetic equations. However, are the results in line with solutions of kinetic equations in the dynamic coordinate system OXYZ? In using transform methods, from $V_o=[V_{ox}V_{oy}V_{oz}]^T$ and $\xi$ = $[X_oY_oZ_o]^T$, one obtains dynamic coordinate system motion 36
parameters, and, in conjunction with that, after one makes a comparison, one, then, has naturally dispelled misgivings. Refering to Fig.1, going through the transformations described below, it is possible to search out the relationship between $O_oX_oY_oZ_o$ and OXYZ.

The first step is to rotate $O_oX_oY_oZ_o$ counterclockwise in a positive direction around $O_oY_o$ an azimuth angle $A_o$ forming $O_oX_o'Y_oZ_o'$. The tangent line $O_oN$ associated with the meridian line of $O_oX_o'$ and the point $O_o$ is duplicated and points north. $O_oZ_o'$, within the horizontal plane at point $O_o$, is perpendicular to the meridian plane and points east.

The second step is to rotate $O_oX_o'Y_oZ_o'$ counterclockwise around $O_oZ_o'$ an angle equal to the latitude $B_o$ forming $O_oX_o''Y_o'Z_o'$. $O_oY_o'$ is parallel to the plane of the equator. $O_oX_o''$ is parallel to the earth's axis of autorotation $EZ_e$.

The third step is, in a situation in which the three axses of $O_oX_o''Y_o'Z_o'$ are maintained in an unchanging direction, to take $O_o$ and translate it to the center of the earth E forming $EX_o''Y_o'Z_o'$.

The fourth step is to take $EX_o''Y_o'Z_o'$ and rotate it counterclockwise through an angle $\omega_e t$ around $EX_o''$, causing $X_o''EY_o'$ to rotate into the meridian plane of point O.

The fifth, sixth, and seventh steps are, respectively, the inverse processes of the first, second, and third steps. Finally, one then takes $O_oX_oY_oZ_o$ and transforms it to a position duplicating OXYZ. Moreover, one obtains the transformation matrix B. It has already been demonstrated that the matrix B is orthogonal. The inverse matrix $B^{-1}$ and the transformed position matrix $B^T$ are equal,

$$\boldsymbol{B}=\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \tag{5.1}$$

This is capable of being written to become the equation below:

$$\boldsymbol{B}=\boldsymbol{B}_4+\cos\omega_e t(\boldsymbol{I}-\boldsymbol{B}_4)+\sin\omega_e t\cdot\boldsymbol{B}_5 \tag{5.2}$$

In the equation, I is a third order unit matrix. $B_4$ and $B_5$ are also both third order matrices:

$$\boldsymbol{B}_4=\begin{bmatrix} b_xb_x & b_xb_y & b_xb_z \\ b_xb_y & b_yb_y & b_yb_z \\ b_xb_z & b_yb_z & b_zb_z \end{bmatrix} \tag{5.3}$$

$$\boldsymbol{B}_5=\begin{bmatrix} 0 & -b_z & b_y \\ b_z & 0 & -b_x \\ -b_y & b_x & 0 \end{bmatrix} \tag{5.4}$$

The relative velocity V in the dynamic coordinate system OXYZ is calculated from the equation

$$\bar{V} = B \cdot V_d \tag{5.5}$$

In this, $V_d=[V_{dx}V_{dy}V_{dz}]^T$ is the relative velocity expressed by the use of (3.5) in $O_oX_oY_oZ_o$. The relative location $=[XYZ]^T$ in OXYZ is also capable of being expressed by the use of matrix B as

$$\eta = B \cdot R - R_1 \tag{5.6}$$

The magnitude r of the distance from the center of the earth r of the current position P of the rocket is

$$r = \sqrt{(X+R_{ox})^2+(Y+R_{oy})^2+(Z+R_{oz})^2} \tag{5.7}$$

Obviously, the invariant inequalities $V^2=V^2_d=V^2_1$ and $r^2=R^2$ are all established. Because of this, it is accurate to make use of the height $h = R-R_1$ by which the rocket leaves the surface of the earth in the inertial coordinate system $O_oX_oY_oZ_o$ to handle the atmospheric constants $\rho/\rho_o$ and $p/p_o$.

The expressions (5.5) and (5.6) for the velocity V and the position $\eta$ in the dynamic coordinate system OXYZ, arrived at from going through transformations of the parameters $V_d$ and $\xi$ in the inertial space $O_oX_oY_oZ_o$, are important results. They clearly show that, in $O_oX_oY_oZ_o$, solving kinetic equations is capable of satisfying various types of requirements. For example, during rocket test flights, ground optical transit theodolites, radar, or other similar tracking and measuring equipment requires V and $\eta$ to act as a reference basis. In these two types of coordinate system, the results from simultaneously solving rocket kinetic equations clearly demonstrate the accuracy of (5.5) and (5.6), verifying, in the coordinate system $O_oX_oY_oZ_o$, that solving the rocket kinetic equations is correct.

## VI. GEOGRAPHICAL COORDINATE COEFFICIENTS

As far as the geographical latitude $B_1$ and the geographical longitude $\lambda$ of the position P for the current location of the rocket at the instant t is concerned, it is also possible to make use of the position R in $O_oX_oY_oZ_o$ to accurately calculate it out. Refering to Fig.1, the geographical latitude $\varphi_x$ can be obtained from

$$\sin\varphi_x = \zeta/R \tag{6.1}$$

In this, R is the distance of the point P from the center of the earth E. $\zeta$ is the vertical height of the point P off the plane of the equator. The geographical latitude is

$$B_1 = t_g^{-1}(t_g\varphi_x/(1-e^2)) \tag{6.2}$$

In reality, this only has meaning on the surface of the earth. However, $\varphi_x$ has significance everywhere. The line connecting the center of the earth E and P intersects with the surface of the earth at point P', In such a case, the distance of the point P' from the center of the earth is

$$R' = R_e\sqrt{1-e^2}/\sqrt{1-e^2\cdot\cos^2\varphi_x} \tag{6.3}$$

As far as the rocket's reaching point P in space at the current instant t is concerned, in terms of the take off point $O_o$'s inertial space span or tensile angle, $J_{1(unclear)}$ is the voyage or course angle. Using the numerical product of the two vectors from the points $O_o$ and P pointing toward the center of the earth E, it is expressed as

$$J_1 = \cos^{-1}\left[\frac{R_1}{R} + \frac{X_0R_{0x}+Y_0R_{0y}+Z_0R_{0z}}{R_1R}\right] \tag{6.4}$$

After going through the time t at the earth's prime meridian, in inertial space, it turns from $EX_e'$ to $EX_e$, that is, about the axis of autorotation of the earth, $EZ_e$ turns through the angle $\omega_e t$. In the same way, OXYZ also, from the originally duplicate location at $O_oX_oY_oZ_o$, rotates through the same angle $\omega_e t$. From Fig.1, one can see that the difference in longitude of the point P relative to point $O_o$ in inertial space is

$$\Delta l = (\lambda - \lambda_0 + \omega_e t). \qquad (6.5)$$

On the spherical surface triangle $EO_oQP'$, respectively making use of cosine theorems and sine theorems, it is possible to solve for an expression for geographical longitude $\lambda$. From the cosine theorem of the course or angle of travel $J_{1(\text{unclear})}$, one has

$$\cos J_l = \cos\left(\frac{\pi}{2} - \varphi_{x0}\right)\cdot\cos\left(\frac{\pi}{2} - \varphi_x\right) - \sin\left(\frac{\pi}{2} - \varphi_{x0}\right) \cdot \sin\left(\frac{\pi}{2} - \varphi_x\right)\cdot\cos\Delta l \qquad (6.6)$$

Because of this, one obtains the formula to calculate the geographical longitude $\lambda$ of the rocket's current position P

$$\cos(\lambda - \lambda_0 + \omega_e t) = (\cos J_l - \sin\varphi_{x0}\cdot\sin\varphi_x)/(\cos\varphi_{x0}\cdot\cos\varphi_x) \qquad (6.7)$$

In this, $\lambda_o$ is the latitude of the $O_o$ point at the instant of the rocket's take off, t = 0 sec. In order to accurately determine the quadrant of $\lambda$, on the same spherical surface triangle $EO_oQP'$, one makes use of the cosine theorem. It is then possible to solve the expression $\sin(\lambda - \lambda_o + \omega_e t)$. From Fig.1, it is possible to see that $\lambda_o$ is also the actual geographical longitude of the origin point O of the dynamic coordinate system OXYZ. The geographical longitude of initiation points or origins $O_o$ and O, $\varphi_{xc} = B_o - \mu_o$. In this,

$$\mu_e = a_e \cdot \sin 2B_e \qquad (6.8)$$

This is perpendicular deviation. However,

$$a_e = (R_e - R_j)/R_e \qquad (6.9)$$

is the rate of flattening of the earth. $R_e$ and $R_j$ are, respectively, the equatorial radius and the polar radius of the earth. The azimuth angle of the current position P is

$$f_l = A_0 + f_1 \qquad (6.10)$$

In this, $f_1$ is the angle of offset or deviation relative to the main plane of the flight $X_oY_oZ_o$ for the current time position of the rocket P, that is, the included angle between $O_oP$ and $X_oO_oP_o$. Obviously,

$$f_1 = \sin^{-1}\left(\frac{Z_0 + R_{0z}}{\sqrt{(X_0 + R_{0x})^2 + (Z_0 + R_{0z})^2}}\right) \qquad (6.11)$$

From this, it is possible to obtain a precise determinatinon of the quadrant of $\lambda$. 38

## CONCLUSIONS

The practical realization of the study and design of the inertial guidance systems of our carrier rockets clearly demonstrates that, in the inertial coordinate system $O_oX_oY_oZ_o$, which is illucidated by this article, the solutions of rocket kinetics and the derivations of various individual formulae are correct. This is particularly true for the motion parameters V and $\eta$ as well as geographical coordinates B and $\lambda$ and is completely in line with the results obtained in the dynamic coordinate system OXYZ from solving kinetic equations.

Making use of the inverse matrix $B^{-1}=B^T$, taking carrier rockets' actual flight course track parameters and transforming them into $O_oX_oY_oZ_o$, the carrying out of checks and tests on the errors of inertial instrumentation and the analysis of other errors in guidance systems must, of necessity, be feasible.

There are a number of representative works[1,2,3,4] and treatises which, in the dynamic coordinate system OXYZ, which rotates following the earth, set up and solve kinetic equations of rockets, handle various types of problems associated with flight force mechanics, calculate aerodynamic forces with relative ease, make observations and measurements as well as tracking rocket motions without the need for transformations. With regard to the extreme importance of motions and errors associated with the study of carrier rocket inertial guidance systems in inertial space as well as inertial instrumentation, the methods studied in this article have value. In particular, writing them down into an article is convenient for use as a reference. In conjunction with that, they have been even more deeply discussed with coworkers.

In the process of the studies in this article, we were aided by Li Zhentao, Wang Guangmin, and other similar comrades. We have opted for the use of several formulae associated with rocket kinetic equations set-up by them in the OXYZ system. We wish to express our sincere thanks to them!

REFERENCES

[1] Kaplan, M. H.: Modern Spacecraft Dynamics & Control. 1976.
[2] Greensite, A. L.: Analysis and Design of Space Vehicle Flight Control Systems, 1970.
[3] Гобатенко,.С.А: Механика Полета, 1969.

[4] [Soviet] A.A. Demiteliyefuxiji (phonetic, possibly Demetrievski); Foreign Ballisitics

[6] Yi Zhaohua, et.al.; Selected Discussions on Celestial Mechanics, 1978

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